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Technical Notes

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Mixed Convection from a Vertical Plate to Power-Law Fluids

Ming-Jer Huang* and Bing-Lin Lin†
National Cheng Kung University, Tainan 70101
Taiwan, Republic of China

Introduction

COMPREHENSIVE reviews of studies on heat transfer over two-dimensional or axisymmetric bodies in non-Newtonian power law fluids have been presented.[1–4] In the present note, we consider mixed convection in power-law fluids along a vertical plate. Two parameters, the buoyancy parameter and the generalized Prandtl number, describe the nonsimilar characteristics of the problem. Here we select the buoyancy effect as the controlling parameter excluding the local coordinate x, and introduce a generalized Prandtl number, Pr_x, as the streamwise parameter. Although the generalized Prandtl number represents the properties of power-law fluids, it also indicates the location from the leading edge of the plate to far downstream.

Analysis

Consider a steady, two-dimensional incompressible, laminar boundary-layer flow of a power-law fluid past a vertical flat plate. By employing the Boussinesq approximations and making use of the power-law viscosity model, the governing equations can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \frac{K}{\rho}\frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where K and n are, respectively, the fluid consistency index and flow index for power-law fluids, and other notations are the conventional ones.

The associated boundary conditions are

$$y = 0;\ u = v = 0,\ T = T_w \quad \text{or} \quad \frac{\partial T}{\partial y} = -\frac{q_w}{k} \tag{4a}$$

$$y \to \infty;\ u \to u_\infty,\ T \to T_\infty \tag{4b}$$

Based on an order-of-magnitude scaling analysis,[5] the thermal boundary layer thickness δ_t can be expressed as

$$\delta_t = x/[Re_x^{1/(n+1)} Pr_x^{1/3} \xi^{1/6}] \tag{5}$$

where $\xi = Pr_x/(1 + Pr_x)$, Re_x is the generalized Reynolds number, and Pr_x is the generalized Prandtl number. The last two quantities are defined as

$$Re_x = \rho u_\infty^{2-n} x^n / K \tag{6}$$

$$Pr_x = (1/\alpha)[(K/\rho)]^{2/(n+1)} x^{(1-n)/(n+1)} u_\infty^{3(n-1)/(n+1)} \tag{7}$$

In addition, by using the following dimensionless form:

$$\eta = y/\delta_t \tag{8}$$

$$f(\xi, \eta) = \psi(x, y)/(\alpha x/\delta_t) \tag{9}$$

$$\theta(\xi, \eta) = (T - T_\infty)/(T_w - T_\infty) \quad \text{for} \quad T_w = \text{const} \tag{10a}$$

$$\theta(\xi, \eta) = (T - T_\infty)/(q_w k/\delta_t) \quad \text{for} \quad q_w = \text{const} \tag{10b}$$

the transformed system of equations can be written as

$$a_1[|f''|^{n-1}f'']' + a_2 ff'' + a_3 f'^2 + a_6\theta = a_5\left(f'\frac{\partial f'}{\partial \xi} - f''\frac{\partial f}{\partial \xi}\right) \tag{11}$$

$$\theta'' + a_2 f\theta' + a_4 f'\theta = a_5\left(f'\frac{\partial \theta}{\partial \xi} - \theta'\frac{\partial f}{\partial \xi}\right) \tag{12}$$

$$f(\xi, 0) = f'(\xi, 0) = 0,\ \theta(\xi, 0) = 1 \quad \text{or} \quad \theta'(\xi, 0) = -1 \tag{13a}$$

$$f'(\xi, \infty) \to (1 - \xi)^{-1/3},\ \theta(\xi, \infty) \to 0 \tag{13b}$$

where primes denote partial differentiation with respect to η, and

$$a_1 = \xi^{(n+1)/2}/(1 - \xi) \tag{14}$$

$$a_2 = 0.5 + (n - 1)\xi/[6(n + 1)] \tag{15}$$

$$a_3 = -(n - 1)\xi/[3(n + 1)] \tag{16}$$

$$a_4 = 0 \quad \text{for} \quad T_w = \text{const} \tag{17a}$$

$$a_4 = 0.5 + (n - 1)\xi/[6(n + 1)] \quad \text{for} \quad q_w = \text{const} \tag{17b}$$

$$a_5 = -(n - 1)\xi(1 - \xi)/(n + 1) \tag{18}$$

$$a_6 = \left\{\frac{\xi}{(1 - \xi)^{(5+n)/[3(n+1)]}\Omega}\right\}^{(n+1)/(1-n)} \quad \text{for} \quad T_w = \text{const} \tag{19a}$$

$$a_6 = \left\{\frac{\xi^{(5+n)/[2(n+1)]}}{(1 - \xi)^{(7+2n)/[3(n+1)]}\Omega}\right\}^{(n+1)/(1-n)} \quad \text{for} \quad q_w = \text{const} \tag{19b}$$

with

$$\Omega = \frac{1}{\alpha}\left(\frac{K}{\rho}\right)^{2/(n+1)} u_\infty^{[3(n-1)]/(n+1)} \left[\frac{u_\infty^2}{g\beta(T_w - T_\infty)}\right]^{(1-n)/(n+1)} \quad \text{for} \quad T_w = \text{const} \tag{20a}$$

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*Professor, Department of Engineering Science.
†Graduate Student, Department of Engineering Science.

$$\Omega = \frac{1}{\alpha}\left(\frac{K}{\rho}\right)^{2/(n+1)} u_\infty^{3(n-1)/(n+1)} \left[\frac{u_\infty\, \alpha^{1/(n-1)}}{g\beta\left(\dfrac{K}{\alpha}\right)^{1/(n-1)} \dfrac{q_w}{k}}\right]^{(1-n)/(n+1)}$$

for $q_w = \text{const}$ (20b)

The buoyancy parameter Ω, defined by Eq. (20), represents the buoyancy effect on the forced convection. When the buoyancy effect increases, this parameter becomes smaller for $n < 1$ and larger for $n > 1$. In other words, the forced convection is dominant for $n < 1$ as $\Omega \to \infty$ and for $n > 1$ as $\Omega \to 0$. This nonsimilar transformation as described above, can be successfully employed to nonNewtonian power-law fluids, except Newtonian fluids ($n = 1$).

The local friction factor $c_f = 2\tau_w/(\rho u_\infty^2)$, and the local Nusselt number $Nu_x = hx/k$ are readily expressed as

$$c_f Re_x^{1/(n+1)}/2 = \xi^{1/2}[f''(\xi, 0)]^n \tag{21}$$

$$Nu_x/Re_x^{1/(n+1)} = \xi^{1/2}(1 - \xi)^{-1/3}[-\theta'(\xi, 0)]$$

for $T_w = \text{const}$ (22a)

$$Nu_x/Re_x^{1/(n+1)} = \xi^{1/2}(1 - \xi)^{-1/3}[\theta(\xi, 0)]^{-1}$$

for $q_w = \text{const}$ (22b)

Results and Discussion

An implicit finite difference scheme was used to solve the system of Eqs. (11–13). The details of the solution procedure are described in Refs. 6 and 7.

The values of local Nusselt number, $Nu_x/Re_x^{1/(n+1)}$, as a function of Pr_x for various values of the buoyancy parameters Ω, and fluid flow indices n, are shown in Figs. 1 and 2 for T_w = constant, and in Figs. 3 and 4 for q_w = constant, respectively. As mentioned earlier, the buoyancy effect gradually increases from pure forced convection; the value of Ω thus decreases from infinity for $n < 1$ and increases from zero for $n > 1$. It is clear from the figures that by increasing the buoyancy effect, the local Nusselt number is increased for all power-law fluids. However, different trends in the variations of the local Nusselt number can be found by comparing Fig. 1 with Fig. 2 or Fig. 3 with Fig. 4. Since the leading edge of the plate corresponds to $Pr_x \to 0$ for $n < 1$ and to $Pr_x \to \infty$ for $n > 1$, the curves of local Nusselt number for $n < 1$ are steadily increased as Pr_x increases for a given value of Ω. However, for $n > 1$, the curves are first decreased and then increased as Pr_x increases. The point of departure from the pure forced convection is closer to the leading edge as the buoyancy effect is increased.

To describe the buoyancy effect on forced convection from a vertical plate to nonNewtonian fluids, we depict the new correlating equations for the local Nusselt number as

$$Nu_{x,M}/Re_x^{1/(n+1)} = Pr_x^{1/3}\left\{a^N + \left[b\cdot\left(\Omega^{(n+1)/(n-1)} \cdot Pr_x^{(4n+2)/3(1-n)}\right)^{1/(3n+1)}\right]^N\right\}^{1/N} \quad \text{for} \quad T_w = \text{const} \tag{23a}$$

$$Nu_{x,M}/Re_x^{1/(n+1)} = Pr_x^{1/3}\left\{\tilde{a}^N + \left[\tilde{b}\cdot\left(\Omega^{(n+1)/(n-1)} \cdot Pr_x^{(5n+4)/3(1-n)}\right)^{1/(3n+2)}\right]^N\right\}^{1/N} \quad \text{for} \quad q_w = \text{const} \tag{23b}$$

where $N = 3e^{n-1}$, $\tilde{b} = 0.63$, and values of a and b are given in Eq. (18) of Shenoy.[8] In addition

$$\tilde{a} = -0.175n^5 + 1.075n^4 - 2.52n^3 + 2.809n^2 - 1.468n + 0.746 \tag{24}$$

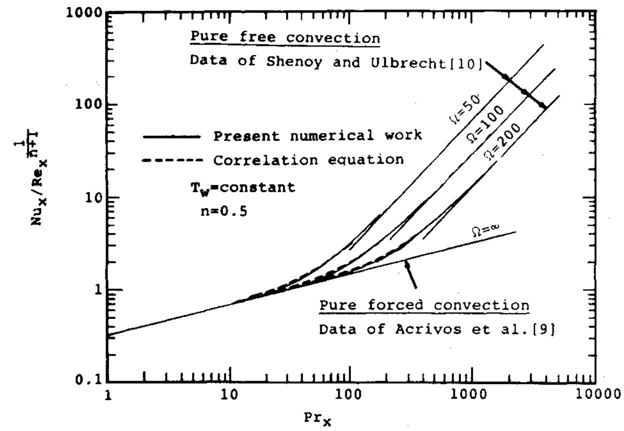

Fig. 1 Local Nusselt numbers vs Pr_x for T_w = const and n = 0.5.

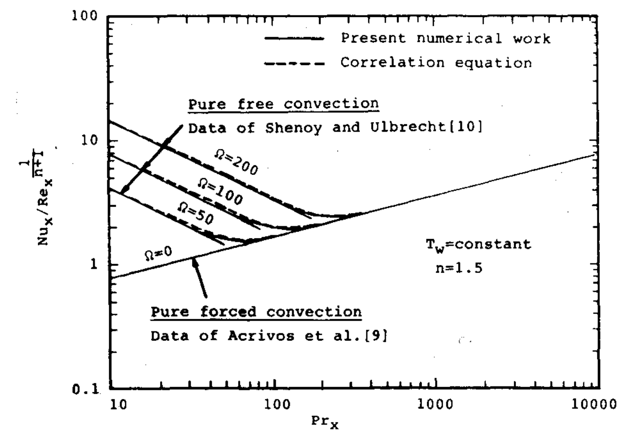

Fig. 2 Local Nusselt numbers vs Pr_x for T_w = const and n = 1.5.

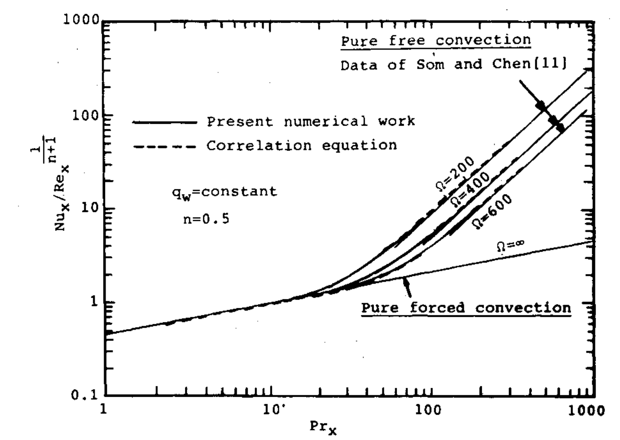

Fig. 3 Local Nusselt numbers vs Pr_x for q_w = const and n = 0.5.

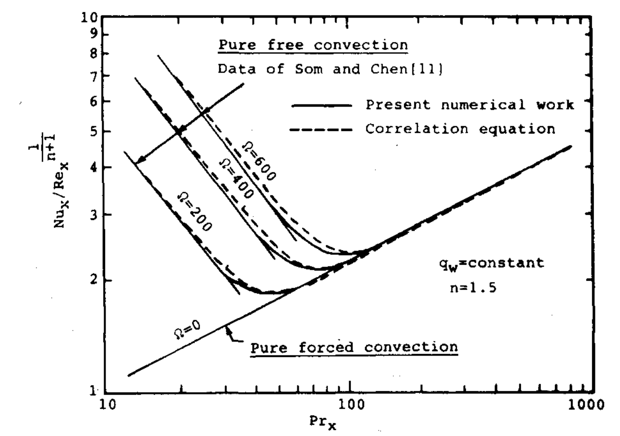

Fig. 4 Local Nusselt numbers vs Pr_x for q_w = const and n = 1.5.

In Figs. 1–4, the solid line represents the present numerical results from Eq. (22) and the dashed line is from the correlation Eq. (23). It can be seen that the present numerical results provide good agreement with the two limiting cases of pure forced[9] and pure free convection.[10,11] Thus, we can be confident of the accuracy of the present numerical scheme. It is noted that the maximum error between the present numerical results and the correlation equation is less than 8%.

Acknowledgment

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View Factors for Perpendicular and Parallel Rectangular Plates

J. R. Ehlert* and T. F. Smith†
University of Iowa, Iowa City, Iowa 52242

Nomenclature

A = area
d = distance between surfaces
F_{1-2} = view factor between surfaces 1 and 2
G = integration function
s = distance
x = distance
y = distance
η = distance
θ = polar angle
ξ = distance

Subscripts
1 = surface 1
2 = surface 2

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*Graduate Student, Department of Mechanical Engineering.
†Professor, Department of Mechanical Engineering. AIAA Member.

Introduction

ANALYSIS of radiant exchange between surfaces separated by a radiatively transparent medium is of importance in several applications, including spacecraft thermal control, electronic thermal control, and building thermal environments. One procedure for performing the analysis is based on the radiosity-irradiation method.[1,2] Fundamental to this method is the calculation of the view factors that describe the radiant exchange between two surfaces. The view factors depend only on geometry of the participating surfaces. Evaluation of view factors continues to be of interest for a wide range of geometries. One geometric configuration that appears frequently is the parallel and perpendicular arrangements of surfaces. Howell[3] presented view factor expressions based on the analysis of Gross et al.[4] for rectangular perpendicular and parallel plates having parallel boundaries. During the use of these view factor expressions for developing a code to verify a three-dimensional discrete-ordinates model based on that of Sánchez and Smith,[5] it was apparent that simplified expressions could be developed. The purpose of this note is to present alternative expressions that are simplified over those available in the literature.

Analysis

Schematic diagrams of perpendicular and parallel surfaces labeled A_1 and A_2 are displayed in Fig. 1. It is of interest to obtain the view factor F_{1-2}. View factor algebra[1,2] could be used to find the required view factors but becomes quite lengthy and tedious as the number of surfaces increases and is subject to errors. The approach of Gross[4] is used to eliminate the need to perform view factor algebra. A notation similar to that of Gross et al.[4] is adopted for developing the view factor expressions.

The view factor is defined by

$$A_1F_{1-2} = \int_{A2}\int_{A1} \frac{\cos\theta_1 \cos\theta_2}{\pi s^2} \, dA_1 \, dA_2 \tag{1}$$

where θ_1 and θ_2 are polar angles for surfaces 1 and 2, respectively. Surface area A_1 has Cartesian coordinates x, y, and surface area A_2 has Cartesian coordinates η, ξ as shown in Fig. 1.

The distance and polar angles for the perpendicular plates are

$$s^2 = x^2 + (y - \eta)^2 + \xi^2 \qquad \cos\theta_1 = \xi/s \qquad \cos\theta_2 = x/s \tag{2}$$

which, when inserted into Eq. (1), give

$$G(x, y, \eta, \xi) = \frac{1}{\pi}\int_\xi\int_\eta\int_y\int_x \cdot \frac{x\xi}{[x^2 + \xi^2 + (y - \eta)^2]^2} \, dx \, dy \, d\eta \, d\xi \tag{3}$$